

EIGENVECTOR LOCALIZATION AS A TOOL TO STUDY SMALL COMMUNITIES IN ONLINE SOCIAL NETWORKS

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We present and discuss a mathematical procedure for identification of small “communities” or segments within large bipartite networks. The procedure is based on spectral analysis of the matrix encoding network structure. The principal tool here is localization of eigenvectors of the matrix, by means of which the relevant network segments become visible. We exemplified our approach by analyzing the data related to product reviewing on Amazon.com. We found several segments, a kind of hybrid communities of densely interlinked reviewers and products, which we were able to meaningfully interpret in terms of the type and thematic categorization of reviewed items. The method provides a complementary approach to other ways of community detection, typically aiming at identification of large network modules.

Keywords: social network; random matrix; internet

1. Introduction

The complexity of our societies is studied by social analysts in various ways. Qualitative inquiries and case studies usually put little emphasis on formalized description, partly to avoid oversimplification, or even trivialization of the phenomena under study. On the other side, sophisticated mathematical procedures are increasingly used in order to grasp complexity in a specific way, as a formalized property of larger systems. One of the branches of the latter stream is represented by the analysis of social networks using mathematical theory of graphs. Our approach adheres precisely to this field of research and yet, it follows slightly different direction than most efforts in contemporary network analysis.

The purpose of this paper is twofold. First, we want to present a specific solution to a rather standard problem of social network analysis, which is identification of communities within complex networks. Second, we want to discuss some alternative perspectives on the concept of “social network”. We suggest that our method might

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provide a suitable tool for empirical research in respective directions, enabling the analyst to determine those “hot spots” within the network that usually escape attention.

To make the wider methodological context of our paper clearer, let us start with some notes on networks and network analysis in contemporary sociology. The use of the term “network” in contemporary sociology vary from loose metaphors [17] to rather specific and technical meanings [23, 83], compatible with the network science as understood in mathematics or physics.

Social network analysis has a complex history, with roots involving the sociometric analysis of Moreno in the beginning of 20th century, the Harvard researchers of the 1930s and 1940s who studied interpersonal configurations and cliques and, finally, the group of anthropologists based in Manchester who, roughly in the same time, instead of emphasizing integration and cohesion as their predecessors, focused on conflict and change, see [83], pp. 7-37. In 1960s, a key turn to mathematization occurred, which gave this field a new impulse and high ambitions. Today, encouraged by the rise of interest in networks in other scientific disciplines, social network analysis is sometimes seen as an approach that may entirely redefine the social sciences, while integrating them into a broader interdisciplinary research program [19]. Formalized analytical procedures hugely contributed to the fact that social network analysis has become firm basis for social science discussions [90]. However, integration of mathematical analytic thinking with sociological imagination is an intricate task. As noted by [31], the application of formalized methods of social network analysis is often marked by neglecting substantive and theoretical sociological consequences. Also, despite the growing popularity of mathematical modeling, qualitative, or ethnographic studies of “network sociality” [92] keep their relevance, hand in hand with quantitative approaches.

Given this complicated background, our aim, in this paper, is rather modest. We want to introduce and illustrate a new mathematical method for identification of small parts of complex networks with higher level of commonalities and for studying their basic formal properties. As an example and possible field of application we have chosen networks of product reviewing on the Amazon.com portal. Here, the simplest possible ties structuring the network are the connections established by two reviews written on the same product. In other words, what reviewers may have in common is the product reviewed by them. The configurations when one product, e.g., a book or a CD, is reviewed by two or more reviewers are frequent, of course, and not much special. But if the same reviewers are similarly connected via some other items too, the situation gets more exciting. We can assume that network segments with higher density of such links represent small communities of reviewers with similar interests. Our first and main objective is to find these small communities.

Identification of such small-size groupings has always been one of the key tasks in social network analysis. Identification of these network segments is an interesting empirical finding in itself. Other times, however, the need to focus on smaller

network segments is rather methodological than substantial or theoretical: for instance, when David and Pinch [22] analyzed the phenomenon of review plagiarism on Amazon.com, they had to “localize” the phenomena in order to make it better graspable in detail. Thus, they had to reduce their sample while focusing on those segments of the vast amount of data available in which reviewed products were “somewhat similar to one another” and thus vulnerable to “recycling” practices they were interested in. This is a characteristic situation: complex networks, including the social ones, are quite often huge, only hardly analyzable in details, with respect to local deviations or little extremities. This is especially true for on-line networks. When studying internet-related network structures, analysts can quickly become overloaded with data and it is difficult to tell what exactly to look at. The urgent question becomes: how to locate tiny islands of relevance in the ocean of data archived on the Internet? We offer a possible mathematical method for precisely such a task – a more flexible and background-sensitive one (a “softer” one, in a way) than those already described and used in the field.

We should also stress at this point that our task differs from the well-studied problem of splitting the network into several modules, which may perhaps overlap, but as an ensemble, they cover the network entirely. This is the case in metabolic networks, to mention just one example [72, 46]. In our case, we want to focus on a few “hot spots”, small communities of interest within the network, leaving all the rest behind.

2. Reviewing networks on Amazon.com as a sociological problem

Before demonstrating the mathematical procedures, let us also briefly mention some sociological contexts of the chosen example. Sociologists have pointed out the increasing importance of the symbolic content of contemporary economics, which is often associated, among others, with users’ or consumers’ active involvement in the complex processes of product evaluation, qualification, and formation [2, 59]. The role of consumers is particularly enhanced by the Internet and by the ways computer technologies shape social networks [15].

A specific and significant part of these processes has been recognized as “peer-production of relevance/accreditation” [8], p. 75 or simply as “reputational economy” [22]. By reviewing or commenting items in on-line shops, classifying and rating them, individual consumers become co-producers of coordinates for others’ economic decision making. They engage in a complex action that cannot be simply grasped in purely economic terms. As noted by [62], p. 322, spaces of E-commerce are characteristic by countless devices creating diversity of forms of encounter between products and consumers [93].

User reviews and comments, for instance, not only serve the purposes of the seller, but also the consumer community, while simultaneously being the means for identity building of reviewers themselves [37]. In-depth study of all these complex phenomena seems crucial for better understanding of contemporary “technological

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economies” [6].

What kind of groupings are we interested in when we try to locate segments of reviewers connected by shared reviewed products? We might be tempted to talk about virtual communities. But these would not be “virtual communities” in the usual sense [78, 91]; and they would not be “online social networks” as typically imagined by social scientists. Both these concepts characteristically refer to groups of people who directly communicate to each other with the help of computer networks – i. e., who know (about) each other and interact by means of on-line forums, instant messaging, or facebook. Our groups of Amazon.com reviewers represent a slightly different kind of entities, though. These people usually do not communicate by addressing each other and quite often they even do not know each other. They do not belong to the group by virtue of intentional interaction with the others, but “merely” by doing similar things in a relatively uncoordinated way: writing reviews on specific products. If [45, 44] drew our attention to the importance of “weak ties” in social networks, i.e., to the significance of ordinary informal acquaintances (in comparison to family ties and formal hierarchies), we could speak here of a kind of “ultra-weak ties”. These ties are “virtual” in the sense that they are not “real enough” in the usual sociological meaning; yet, they are materialized and articulated - although not by the reviewers themselves only. We can clearly see the connections on the Amazon.com web pages: the reviews of these people are listed together, accompanying the respective item in the catalog. Moreover, the reviewers do not become members of this community completely unintentionally, but by means of quite intentional and personal act of assessing the product and writing the review. They create the community by highly mediated interactions, as if “by the way”, together and via the technology of on-line shopping.

In the following section we present mathematical tools for identification, representation and elementary description of precisely such communities. The proposed procedures may have a value especially in relation to subsequent sociological analysis of these local anomalies, as its precondition.

3. Finding small communities in networks

3.1. *Motivation*

The problem of identification parts of the network bearing some relevant structural information, can be relatively easily formulated in mathematical terms. The methodological problem is, which one of the variety of possible mathematical formulations of community detection is suitable for given purpose. Let us stress that we neither aim at improving the existing schemes nor present an algorithm which should compete with the established ones. Instead, we are bringing an alternative scheme which reveals structures, not covered by other schemes of community detection. That is why we not only present a description of the method and its application to one real-world example, but also spend time putting it into a wider context of sociological thinking.

3.2. Background

For a long time, the standard way of mathematical modeling of social networks [90, 23] was the “classical” theory of random graphs [11, 24] initiated by the work by Erdős and Rényi [32]. However, in the last decade a new class of networks became studied and the name “complex networks” became common denomination for them [4, 1, 88, 27, 12, 28, 70]. Compared with the “classical” models of random networks, they grasp much better the networks found in reality and at the same time their models are much more involved than bare random dropping of edges as in the Erdős-Rényi model. The most immediate characteristics common to the complex networks is their degree distributions with power-law tails [4].

The strong inhomogeneity of complex networks, implicit in their degree distribution, changes many aspects of their behavior. In the context of our work, new approaches for finding the communities become relevant. While the methods for determining cliques, k -cliques and motifs [90, 23] work well if the zero-hypothesis on the network structure is the Erdős-Rényi random graph, methods better suited for complex networks were developed [54, 33, 68, 69, 77, 76, 72, 40, 16, 46, 39, 5, 57, 38, 82, 61, 58, 5, 56]. The central quantity for majority of them is the modularity measure Q , which is to be made maximal. This is achieved by various optimization algorithms.

Here we will rely on the method of describing the global properties of networks using the spectral theory of graphs [20]. It deals with eigenvalues and eigenvectors of various matrices representing the graph structure, which are the adjacency matrix, Laplacian and more. It was already used for finding clusters or communities in networks through the properties of eigenvectors corresponding to the second largest eigenvalue [67, 16, 21, 25]. In one step it gives the best partitioning of the network into two modules and repeating the algorithm recursively, the communities are found. Our approach is different, though. It is similar in spirit to the analysis of covariance matrices in finance [73, 75], where economic sectors are attributed to eigenvectors corresponding to the second, the third, etc. largest eigenvalue.

The first level of understanding spectral properties of a random matrix comprises the knowledge of the density of eigenvalues. The second involves the localization properties of the eigenvectors. It is the latter that is central for our approach.

Let us say first a few words on the eigenvalue density. Spectra of “classical” random graphs, like the Erdős-Rényi model, are closely related to “classical” models of random matrices [64]. The typical shape of the eigenvalue density is the Wigner semi-circle with sharp edges, with the largest eigenvalue split far off from the bulk of all other eigenvalues. The first complication arising in the spectrum of a random graph is the sparseness of the adjacency matrix, which leads to the emergence of Lifschitz tails. This appears already in the Erdős-Rényi model. Despite considerable effort [79], the Lifschitz tail in ER graph is still not known in all details. Asymptotic formula was obtained by several approaches, showing that the density of states is non-zero at arbitrarily large eigenvalues and it decays faster than any power law

[79, 84].

It was soon realized that complex networks, characterized by power-law degree distributions, have also non-standard spectral properties [34, 42, 29, 30, 33, 71, 53, 35, 51, 36, 85, 21, 14, 94, 26, 86]. First, there is a cusp in the middle part of the density of eigenvalues, and second, perhaps more importantly, the tail of the eigenvalue density seems to be described by a power law [34, 42]. Numerical diagonalization on toy models [29, 30] as well as some analytical estimates confirmed power-law tails in the density of states. The replica trick [80, 65], as well as the cavity method [30, 81] were later adapted for scale free networks. It was found that the spectrum has a power-law tail characterized by the exponent $2\gamma - 1$ related to the degree exponent γ of the network. Further improvements of the method were introduced recently [50, 10].

As we shall see, our method is similar to those used in the study of covariance matrices of stock-market fluctuations [41, 55, 73, 74, 75, 89, 3, 48, 13]. They are modeled as random matrices of the form $M M^T$, where M is a random rectangular matrix. The density of states has the Marčenko-Pastur form [63] with sharp edges, which are smeared out into Lifschitz tails if the matrix is sparse [66].

Most attention was paid to the states in the tail, i. e. located beyond the edge of the Marčenko-Pastur density and below the maximum eigenvalue, which is always split off. These states are supposed to carry the non-trivial information about the stock market and, indeed, the shape of the corresponding eigenvectors was used to identify business sectors. It was supposed that the eigenvectors were localized on items within specific sector [43, 75, 52]. More sophisticated approaches were also developed [89].

Our method owes largely to the spectral analysis of covariance matrices. However, we improve these approaches by systematic use of the quantitative measure of localization of the eigenvectors, which is the inverse participation ratio. In an intuitive manner, similar approach was already used in the analysis of gene coexpression data [49]. Within this approach, we do not aim at factoring the entire network into some number of modules, or communities, which may or may not be overlapping, but in any case covering, as an ensemble, the whole network. Instead, we want to find small parts of the network which differ structurally from the rest. We may also describe our approach as “contrast coloring” of the network, which makes certain relevant parts visible against the irrelevant background.

3.3. *Spectral analysis of matrices encoding the structure*

Our analysis will be devoted to bipartite graphs. There are two types of nodes, making up sets \mathcal{R} and \mathcal{I} . Anticipating our application to the Amazon.com network, we think of members of \mathcal{R} as reviewers and members of \mathcal{I} as items to be reviewed. All information on the network structure is contained in the adjacency matrix M with elements $M_{ri} \in \{0, 1\}$. The out-degree of node $r \in \mathcal{R}$ is $k_r = \sum_i M_{ri}$, the in-degree of the node $i \in \mathcal{I}$ is $k_i = \sum_r M_{ri}$.

In bipartite graph, the spectral properties are deduced from the contracted matrices $B = M \cdot M^T$ and $C = M^T \cdot M$. The interpretation of these matrices is obvious; e. g. B_{rs} tells us how many neighbors the nodes r and s have in common. Similar construction is used frequently in bipartite networks. As an example, let us cite the network of tag co-occurrence in the analysis of collaborative tagging systems [18] or recommendation algorithms investigated in [95].

In order to partially separate the effects of the network structure from the influence of degree distribution, we rescale the matrix elements by the product of square roots of the out-degrees. This way, we get the matrix

$$A_{rs} = \frac{B_{rs}}{\sqrt{k_r k_s}} \quad (1)$$

with all diagonal elements equal to 1. We can also be more explicit and write $A_{rs} = (\sum_i M_{ri} M_{si}) / \sqrt{(\sum_i M_{ri})(\sum_i M_{si})}$. Obviously, the matrix A is symmetric.

The matrix A is then diagonalized. Let us see what information can be extracted from the eigenvalues and eigenvectors. First, for any square $N \times N$ matrix D encoding the structure of a graph we can interpret the traces $\frac{1}{N} \text{Tr} D^k$ as density of circles of length k . This number is equal to the k -th moment of the density of eigenvalues of D . In our case, the role of D is assumed by the contracted matrix B and the k -th moment of B expresses the density of cycles of length $2k$ on the bipartite graph. If we use the matrix A instead, the moments of the spectrum are related to the density of weighted cycles. Each time the cycle goes through the vertex $r \in \mathcal{R}$, it assumes the weight $1/k_r$. Therefore, cycles connecting vertexes with large degree are counted with lower weight. This is just what we want here: to put accent on peripheral, less-connected areas of the network, rather than on the hubs. If we did not rescale the matrix as in (1), the weight of the hubs, or strongly-connected nodes in general, would overshadow the major part of the network, where the small communities may lie hidden.

We expect that the spectrum has power-law tail. Indeed, it will be confirmed in the specific example of Amazon.com, which we shall show later. The power-law tail implies that the density of cycles beyond certain length diverges. In terms of the limit $N \rightarrow \infty$ it means that the number of such cycles increases faster than linearly with N . The exponent of the power-law tail tells us what is the threshold for the cycle length, beyond which the cycles are anomalously frequent compared to the Erdős-Rényi graph.

What does all this mean for the problem of finding small compact communities? If we for example use the method of cliques or k -cliques, we tacitly assume that the “background” network does not contain many of these cliques by pure chance. But if, for example, the tail of the spectrum of D has exponent 4, the third moment diverges, which means that there are extremely many triangles. No triangle, or community of size 3, can therefore be considered as informationally relevant. That is why we consider the information on the spectrum of the network an important auxiliary information.

The new algorithm we propose for finding small compact communities relies on the properties of the eigenvectors. Let us denote $e_{\lambda r}$ the r -th element of the eigenvector of the matrix A , corresponding to the eigenvalue λ . To study the localization, we need to calculate the inverse participation ratio (IPR) defined as

$$q^{-1}(\lambda) = \sum_{r=1}^N (e_{\lambda r})^4 \quad (2)$$

where normalization $\sum_{r=1}^N (e_{\lambda r})^2 = 1$ is assumed.

While IPR says quantitatively to which extent an eigenvector is localized, this information alone is not sufficient, if we want to draw the distinction between localized and extended states. First, it makes no sense to ask, if a particular vector is localized, as opposed to extended, or not. What does make sense, however, is the question whether the states around certain eigenvalue are localized. The way to establish that fact is by finite-size analysis. Indeed, if N is the dimension of the vector space we work with, then

$$q^{-1}(\lambda) \sim \begin{cases} O(1), & N \rightarrow \infty \text{ localized state} \\ O(N^{-1}), & N \rightarrow \infty \text{ extended state} . \end{cases} \quad (3)$$

Second, also the shape of the density of eigenvalues changes with the system size. When we increase N , the spectrum broadens. In the textbook example of Erdős-Rényi graph, the spectrum has sharp band edges. The edge of ER spectrum moves as $N^{1/2}$ when N grows and if we compare the IPR at different system sizes, we must measure the eigenvalues relative to the band edge. So, to compare the behavior at different sizes, we take random subset of the network, containing N_{sub} nodes. Typically, we choose $N_{\text{sub}} = N/2$. Then, we plot the density $D(\lambda)$ of eigenvalues for both original network and the density $D_{\text{sub}}(\lambda)$ for the random subset. The densities are rescaled by the factor s , the value of which is found empirically so that the data for $D(\lambda)$ and $D_{\text{sub}}(1 + (\lambda - 1)s)$ overlap as much as possible. The form of this rescaling involves the shift of the eigenvalues by 1, because the matrix A has spectrum centered around the value $\lambda = A_{rr} = 1$. With s found, we plot the IPR for the network and the subset, with the same rescaling as used for the eigenvalues density. The regions, where we observe that $q^{-1}(\lambda)$ remains roughly the same for the network and its subset, are the candidate areas where the localized states are to be found.

We continue the procedure by determining the eigenvectors with largest q^{-1} within the areas of localized states. The elements of these vectors tell us what nodes of the network belong to the small community. To this end, we fix a threshold T and retain only those nodes $r \in \mathcal{R}$ for which the elements exceed the threshold in absolute value, $|e_{\lambda r}| > T$. We do not propose any exact method for fixing T . For the sake of consistency, T must be chosen so that the number of nodes retained is roughly $1/q^{-1}$. In practical applications we observed the number of retained nodes when T was gradually decreased from $T = 1$. At certain crossover value of T we saw

that the number of nodes suddenly started increasing substantially to much larger values than $1/q^{-1}$. So, we fixed T somewhat below this crossover. We believe that this procedure could be made automatic by a software implementation, but we did not do that.

Let us make an important remark at this point. Clearly, we can find some localized states also in a randomized version of the network. These states are results of pure chance and do not bear significant information. Therefore, we cannot exclude that also in the true empirical network, some of the localized states occur just accidentally and thus some of the clusters found are spurious. The choice of the threshold T only cannot discriminate between the true and the spurious clusters. However, looking at the dependence of IPR on eigenvalue for the true and the randomized network (as will be seen later in Fig. 4) we can see the regions where IPR is large and differs markedly between true and randomized networks. The localized states found in these regions (in Fig. 4 it is near the lower edge of the density of states) correspond to clusters that are non-random and do bear relevant information.

This way we find those vertexes $r \in \mathcal{R}$, which form the community $\mathcal{C}_{\mathcal{R}} = \{r \in \mathcal{R} : |e_{\lambda r}| > T\}$. Next, we proceed by finding those $i \in \mathcal{I}$ which are connected to them. Here we can distinguish two levels. First, a vertex $i \in \mathcal{I}$ can be connected to at least two different vertexes from $\mathcal{C}_{\mathcal{R}}$. Then, we say that it belongs to the connectors of the community, $\mathcal{C}_{\mathcal{I}}^{\text{con}} = \{i \in \mathcal{I} : \exists r, s \in \mathcal{C}_{\mathcal{R}} : r \neq s \wedge M_{si} = M_{ri} = 1\}$. Further, those $i \in \mathcal{I}$ which are connected to just one vertex of $\mathcal{C}_{\mathcal{R}}$ form a more weakly bound part of the community, which we call cloud, $\mathcal{C}_{\mathcal{I}}^{\text{cloud}} = \{i \in \mathcal{I} : \exists r \in \mathcal{C}_{\mathcal{R}} \setminus \mathcal{C}_{\mathcal{I}}^{\text{con}} : \wedge M_{ri} = 1\}$. We can explicitly see the asymmetry in constructing the community. This is due to the fact that we focused on the diagonalization of the contraction matrix acting in the space \mathcal{R} . The procedure can be, of course, performed also in the opposite direction, diagonalizing the contraction on \mathcal{I} . Both ways are equally justified on the formal level. The choice should be dictated by practical reasons and by the interpretation we want to draw from the data in any specific application.

To sum up, our procedure for finding small communities in bipartite networks consists in the following steps.

1. Diagonalize the matrix A , $A_{rs} = (\sum_i M_{ri} M_{si}) / \sqrt{(\sum_i M_{ri})(\sum_i M_{si})}$. The output is the density of states $D(\lambda)$ and the inverse participation ratio $q^{-1}(\lambda)$.
2. Do the same for random subset of the network, containing half of the nodes, find the proper rescaling factor s , so that rescaled density of states for the network and the subset coincide. By rescaling the IPR using the same factor s , determine the regions, in which localized states are to be found.
3. Within the localized region, find the eigenvectors with highest IPR.

4. For each of the eigenvectors found, determine the threshold T and establish the set $\mathcal{C}_{\mathcal{R}}$ of nodes r , for which $|e_{\lambda r}| > T$. This set is the projection of the community to the set \mathcal{R} .

5. Find the connector and cloud components of the community on the side of the set \mathcal{I} .

4. An example: Reviewing networks on Amazon.com

4.1. *Basic structural features*

The e-commerce site Amazon.com is one of the oldest and best known on the WWW. It has a very rich internal structure, but the user usually sees only a small part relevant to the service requested in a particular moment. As already announced, we shall investigate one aspect of the Amazon.com trading, namely the network made up of connections between the items to be sold and the reviewers who have written reports on these items.

This network is a bipartite graph, with items $i = 1, 2, \dots, N_{\text{itm}}$ on one side and reviewers $r = 1, 2, \dots, N_{\text{rev}}$ on the other side. The sets of vertexes \mathcal{R} and \mathcal{I} introduced in the methodical section above, correspond to the sets of reviewers and items, respectively.

The reviews written are edges connecting these two sets. The structure of the network can be uniquely described by the matrix M , where the element M_{ri} equals 1 if the reviewer r wrote a review on item i , and 0 otherwise.

The data were downloaded using a very simple crawler in the period from 28 July 2005 to 27 September 2005. First, a list of total $N_{\text{all}} = 1\,714\,512$ reviewers was downloaded; at that time the list containing all Amazon reviewers was accessible through the web. (It is no more so.) The list was naturally ordered by the rank Amazon assigns to each reviewer. On average, reviewers with higher rank have written more reviews, but there are exceptions. For example, at the time of data collection, the No. 1 reviewer, Harriet Klausner, had written 9581 reviews, while the No. 2, Lawrence M. Bernabo, 10603 reviews. This suggests that it is not only quantity but also quality which counts when Amazon ranks their reviewers. We do not touch here the obvious question how the most prolific reviewers do manage reading and reviewing several books per day, throughout many years. As we investigate only structural features here, these problems are left aside.

In the next step, we went through the reviewers' list, from the top rank downwards. We looked only at about 10^5 first reviewers and stopped there, as we considered the sample sufficiently representative. The remaining reviewers are only occasional writers, contributing by one or at most a few reviews. For each reviewer we found all reviews written by her or him and registered the name of the item reviewed (mostly books and CD's, of course, but in general all kinds of goods do appear) as well as some other details about the review. In total, we examined 99 622

reviewers who wrote 2 036 091 reviews on 645 056 items.

4.2. Degree distributions

The simplest and most accessible local property of the network is the degree distribution. In the list of reviewers we put down also the reported number of reviews written by the particular person. We neglected the possible error in this number due to various inconsistencies. We believe that the random discrepancies between the number of reviews reported and number of reviews which can actually be found in the system do not influence the statistics in any significant measure. We show the distribution as out-degree distribution in Fig. 1. We can observe clear power-law dependence except for the few highest degrees. The exponent fitted is $\gamma_{\text{out}} \simeq 2.2$.

Similarly we can extract the in-degree distribution from the list of reviews. The statistics of the number of reviews per item is also shown in Fig.1 and a power-law dependence is found again. The corresponding exponent is now $\gamma_{\text{in}} \simeq 2.35$.

The power distribution is by no means surprising, in view of the vast literature on complex networks. The data provide a clear check that Amazon.com also belongs to the class of networks with power-law degree distribution.

4.3. Distribution of eigenvalues

Now we are in a position to calculate the contraction matrix A acting on the set of reviewers, and diagonalize it. As an additional study, we compare the results with randomized version of the reviewer-item network. This way we discriminate between the influence of the network structure and genuinely random factors.

To this end, we reshuffle the edges in the reviewer-item graph, while keeping the degrees of all vertexes unchanged. The matrix M is replaced by M^R and, correspondingly, the matrix A is replaced by A^R . Again, we can write $A_{rs}^R = (\sum_i M_{ri}^R M_{si}^R) / \sqrt{k_r k_s}$. The only information on the network structure retained here is the order sequence. As we showed in the last section, it obeys a power law, so the features found in analyzing A^R are entirely due to power-law degree distribution, but without further structural details.

We diagonalize the matrices A and A^R . Their eigenvalues $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_N$ are accumulated around the value $\lambda = 1$, which corresponds to the uniform diagonal value of both the true and the randomized matrices. The distributions are plotted in Figures 2, and 3. Let us describe now what we can see here.

In Fig. 2 we plot the histogram of the eigenvalues of the matrix A . Most of them fall within the interval $\lambda \in [0, 3]$, with sharp maximum in the eigenvalue density at $\lambda = 1$. The eigenvalues density is much smaller for $\lambda > 3$ and we show their positions as separate ticks. Although the notions “bulk” and “tail” are not very precise here, we shall use them pragmatically, calling bulk the part with $\lambda \lesssim 3$ and tail the part with $\lambda \gtrsim 3$.

In Fig. 2 we can also see the spectrum of the randomized matrix A^R . The power-law distribution of degrees is preserved. In the spectrum, we can observe certain

remarkable changes. In the bulk of the density of states, as shown in the inset of Fig. 2, the spectrum of the reshuffled network lacks the characteristic tip at the value $\lambda = 1$ and its shape at the lower end of the spectrum is quite different. Most importantly, a sharp band edge develops. On the other hand, at the upper tail of the density of states, the changes are of minor importance.

In Fig. 3 we can compare the behavior of integrated density of states, $D^>(\lambda) = \sum_{i, \lambda_i > \lambda} \frac{1}{N}$ in the region of large eigenvalues. For the original matrix A we observe a power-law decay in the tail $D^>(\lambda) \sim (\lambda - 1)^{-\tau}$ with $\tau \simeq 2$. For the matrix A^R the tail is again quite reasonably fitted on a power law, but with larger exponent. Let us recall that the divergence of the moments of the eigenvalue density is related to the statistics of cycles on the network. For the reshuffled network, the divergence occurs at higher moments, therefore at cycle lengths longer than in the original network. This effect seems to be a tiny one, but this is just a subtle structural difference which goes beyond the bare degree distribution. In short, the Amazon network has many more short loops than how many could be expected knowing only its degree sequence. This suggests the presence of small self-reinforcing communities. Although we do not see them yet at this stage, we can perceive their existence through the density of states of the matrix A .

Interestingly, similar conclusions about small communities were reached in the study of collaborative tagging systems [18], where two-node correlations were calculated in order to estimate the quantity of non-randomness, or semantic information content.

4.4. *Localization*

Having investigated the eigenvalues, let us now turn to the properties of the eigenvectors. We show in Fig. 4 how the IPR depends on the eigenvalue. For the matrix A we can see larger localization around the center of the spectrum at $\lambda = 1$. Farther from the center the localization is weaker, but it increases again at the tails, more strongly at the lower tail, while more gradually at the upper tail. Note also some isolated highly localized states in the bulk of the eigenvalue distribution.

Now we compare the results with the random subset of $N_{\text{sub}} = 5000$. We found that the density of eigenvalues coincides very well if we choose the scaling factor $s = \sqrt[3]{2} = \sqrt[3]{N/N_{\text{sub}}}$. With the same scaling we plot the IPR in Fig. 5. We can see that the absence of a clearcut band edge is complemented by the absence of any region of localized states at the upper end of the spectrum. The lower end does show localized states, though. Therefore, the candidates for compact communities are to be found close to the lower end of the spectrum. In the next section we describe what we have found there.

5. *Finding and interpreting the communities*

As we have said, the most localized states are the candidates for small and densely interlinked communities of reviewers. We counted as members of the community

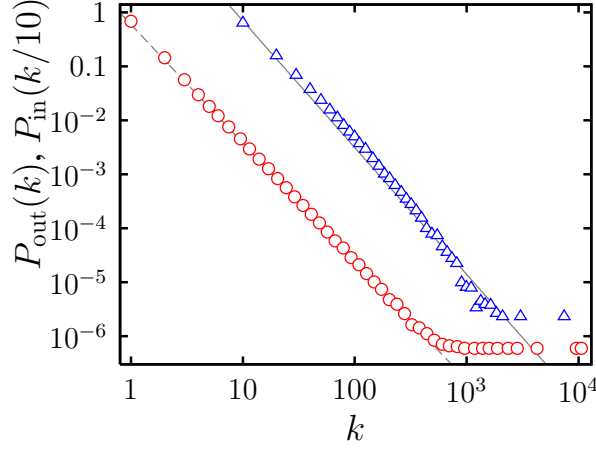


Fig. 1. Degree distribution of the bipartite reviewer-product network on Amazon.com. Circles indicate the data for out-degree (reviews per reviewer), triangles for in-degree (reviews per item). The latter data are shifted rightwards by one decade for better visibility. The lines are the power laws $\propto k^{-2.2}$ (dashed line) and $\propto k^{-2.35}$ (solid line).

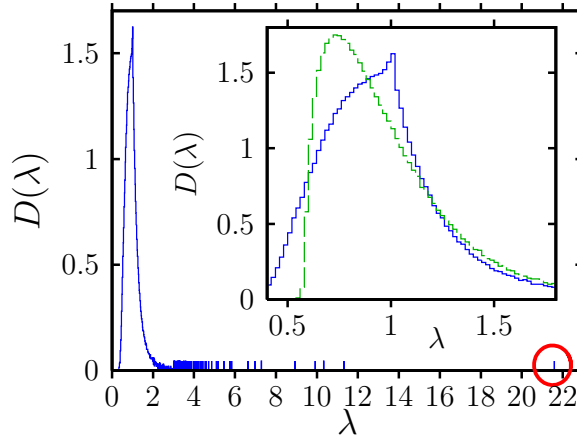


Fig. 2. Distribution of eigenvalues of the reviewer-reviewer matrix. The size of the segment is $N = 10000$. For $\lambda < 3$ the distribution is plotted as a histogram, while the larger eigenvalues, $\lambda > 3$ are shown as individual vertical ticks. The largest eigenvalue is indicated by the circle. In the inset we show the detail of the central part of the same plot. Also in the inset, the dashed (green in color) line is the distribution of eigenvalues of the matrix obtained by reshuffling the reviewer-item graph.

only those reviewers, whose element in the eigenvector was larger than a threshold, $|e_{\lambda r}| > T$. The value of the threshold T was found by trial-and-error, so that all relevant nodes, on which localization appears, were kept, while the remaining ones,

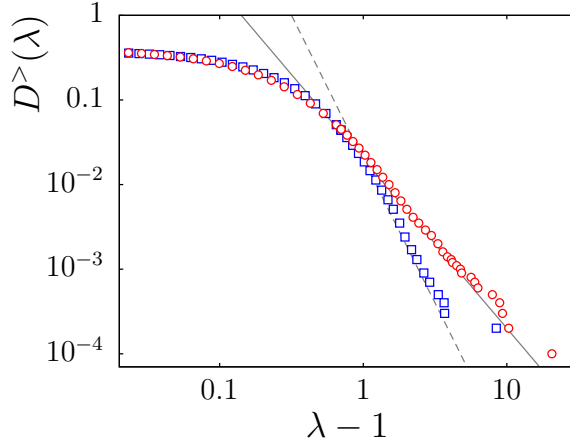


Fig. 3. Detail of the upper part of the distribution of eigenvalues. The behavior is observed using the integrated density of eigenvalues. Circles correspond to the original reviewer-reviewer matrix with $N = 10000$, the triangles correspond to the same matrix subject to permutation of all its elements. The full line is the power $\propto (\lambda - 1)^{-2}$, the dashed line is the power $\propto (\lambda - 1)^{-3.4}$.

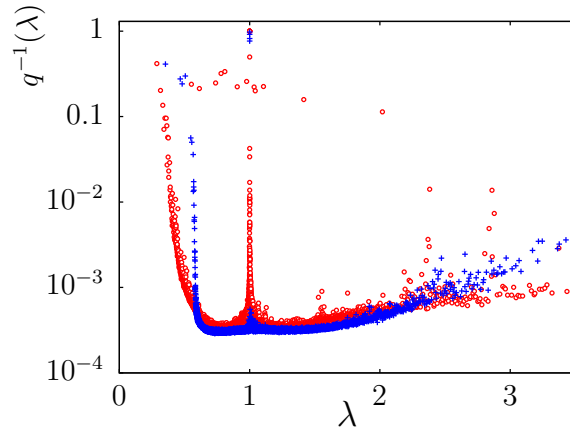


Fig. 4. Inverse participation ratio for the reviewer-reviewer matrix with $N = 10000$ (O), and the same for matrix obtained by reshuffling the reviewer-item graph (+). Each point denotes the IPR for the eigenvector corresponding to the indicated eigenvalue λ .

interpreted as a noisy neighborhood, were left out. This adjustment of thresholds also indicates that the borders of the communities found in this way are not sharp. In our set of 10^4 reviewers, the number of communities which can be considered as well-localized is about $\simeq 10$. We were able to explicitly draw and interpret 7 communities. With average size of the communities around 6 people, the fraction of reviewers in small compact communities can be estimated to about 0.5 per cent. In

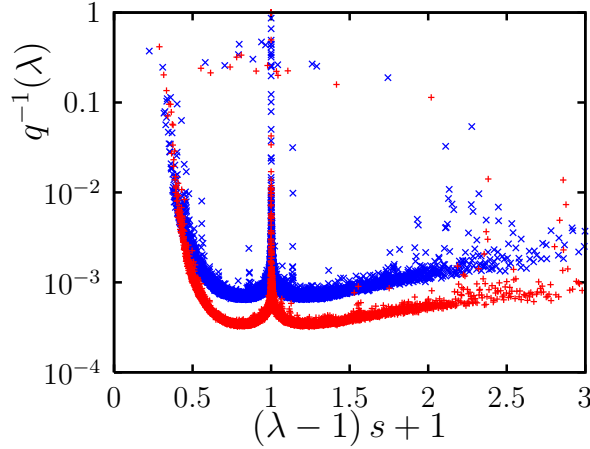


Fig. 5. Inverse participation ratio for the reviewer-reviewer matrix. The horizontal axis is rescaled by the factor s explained in the text. We show the data for the matrix with $N = 10000$ ($+$, $s = 1$), and for the random subset with $N_{\text{sub}} = 5000$ of the same matrix (\times , $s = \sqrt[3]{2}$). Each point denotes the IPR for the eigenvector corresponding to the indicated eigenvalue λ .

other words, we have been able to find relatively rare cases when fractional segments of the network display anomalously high density of mutual links. However, we expect that this fraction would rapidly grow if more reviewers are included from the top of the Amazon list downwards. From this point of view the small percentage of the reviewers in small communities is partly an artifact due to the choice of the reviewers starting from the top of the list of the most productive Amazon.com reviewers.

Now, let us look at several specific examples of the communities found. The first example of such a small grouping is shown in Fig. 6. (In this case we took the 5th most localized vector, $q^{-1} = 0.095675$, corresponding eigenvalue $\lambda = 0.359$, and the threshold was taken as $T = 0.2$.) The items reviewed by the reviewers within the community found in this way are of two types. First, there are those reviewed by at least two reviewers from the community. These items keep the community together and we call them “connectors”. We show them in Fig. 6 linked to their corresponding reviewers. However, one should note that the reviewers themselves play the role of “connectors” for the items, to the same extent as the items are “connectors” for the reviewers. Second, there are items reviewed by only one reviewer of the community. These items form a kind of “cloud” around the core of the network segments. We do not show the “cloud” in our figures, but we shall discuss its meaning later.

What are the product-connectors in the given community (Fig. 6)? We can see that the maximum number of reviewers for one item is 4 and it holds for two audio recordings: “The Beatles (The White Album)” and “Abbey Road” also by Beatles. Thus, the core of the community is kept together by one of the most popular music bands ever. The remaining items are thematically close. They refer to other records

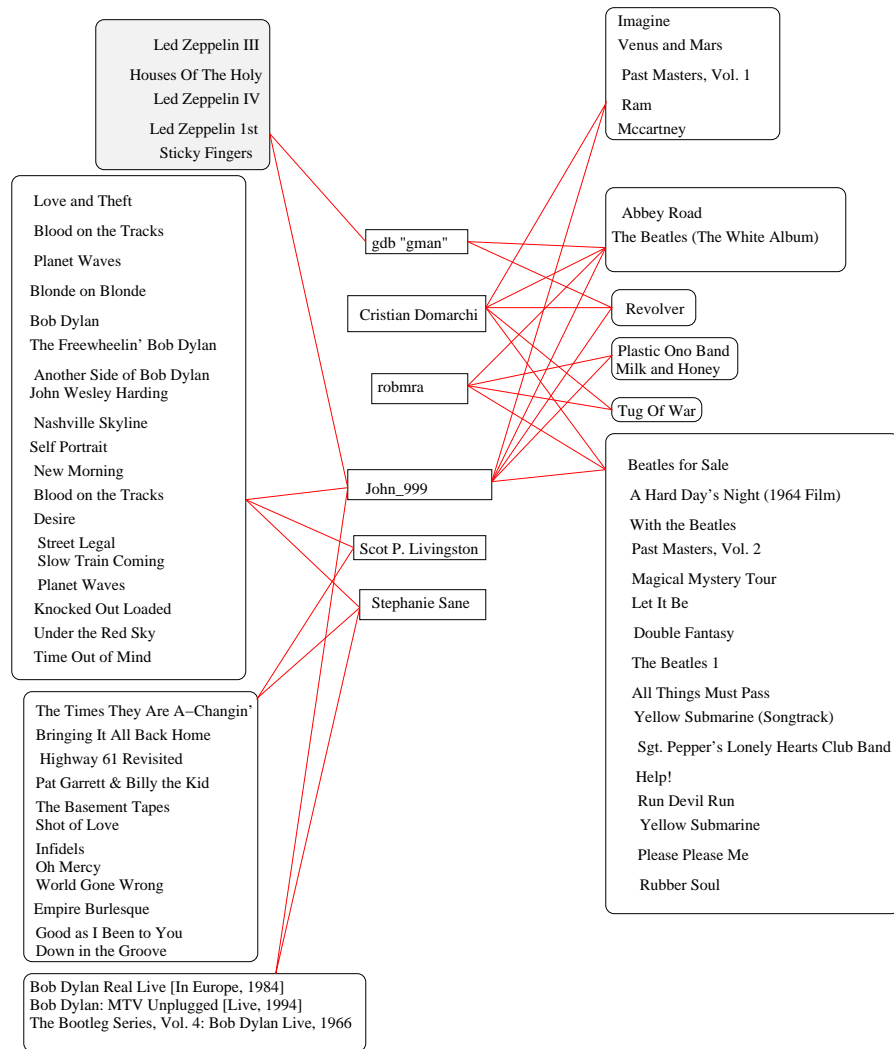
16 *Localization in social networks*

Fig. 6. The “pop-music” community in the network producing a very localized eigenvector of the matrix A . In the middle, code-names of the reviewers, on the right, recordings by The Beatles (mostly as a band, some other by individual members), on the left, recordings by Bob Dylan, with exception of the shaded box which contains four times music by Led Zeppelin and once Rolling Stones.

by Beatles and also by Beatles ex-members, or to the music of Bob Dylan. (Ex-)Beatles and Dylan cover about a half of the items each. The only exception is a small set of five recordings of other pop-classics, namely four of Led Zeppelin and one of Rolling Stones. In short, all items fall into the range of notoriously known pop-music stars. It is interesting that this characteristic does not concern

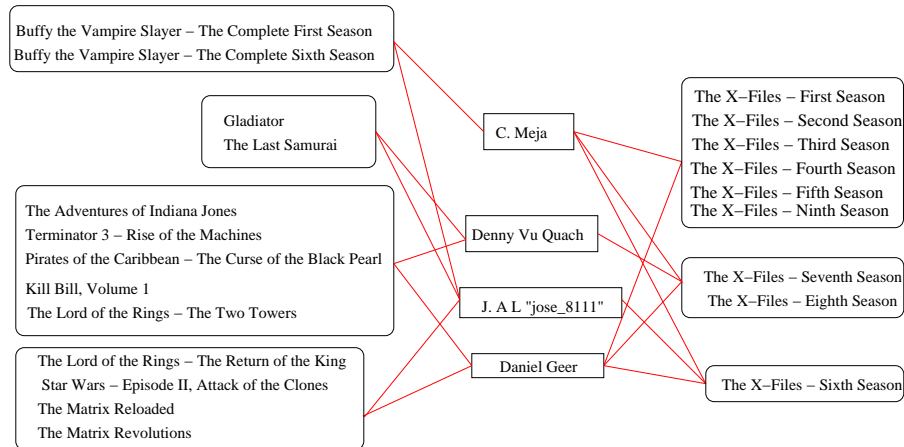


Fig. 7. The “pop-movie” community in the network producing a very localized eigenvector of the matrix A . In the middle, code-names of the reviewers, on the right, the X-Files series, on the left, other popular movies.

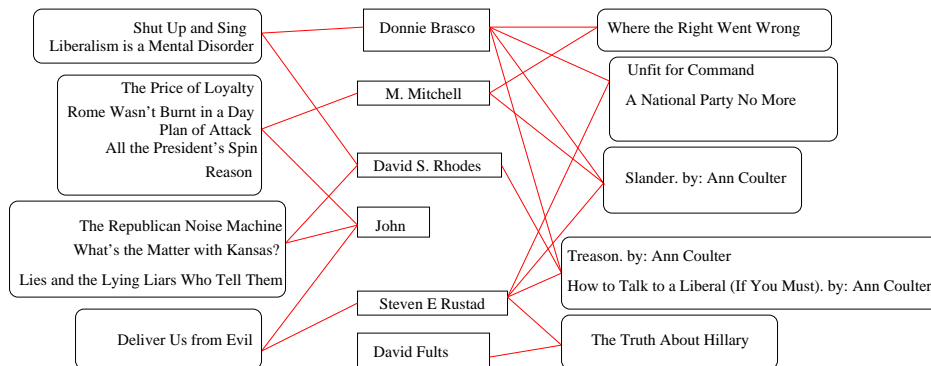


Fig. 8. The “pop-politics” community in the network producing a very localized eigenvector of the matrix A . In the middle, code-names of the reviewers, on the left and right, books treating mainly the clash of (neo-) conservatives versus liberals in the USA. Note that Ann Coulter is the most prominent book author in this community.

the connecting items only, but majority of all other reviews by the members of the community (not included in the graph). Thus, not only the connectors, but also the “cloud” bears the same characteristics.

Therefore, the interest of these reviewers lies, in general, within a rather narrow scope determined by the pair Dylan-Beatles, with some small excursions farther into mainstream pop-music, similar to the small “Led Zeppelin” set in Fig. 6. For example, the reviewer gdb has also written on CD’s by U2 and David Bowie, while the “cloud” reviews by Cristian Domarchi (not listed in Fig. 6) pertain only to other

recordings by ex-Beatles plus one book; among all the 6 reviewers, only Stephanie Sane shows interests which go clearly beyond the Dylan-Beatles repertoire, reviewing a good deal of books, mostly mystery and detective fiction.

A similar picture is provided by the analysis of other communities. Let us very briefly describe two more of them.

The first one (Fig. 7) belongs to another pop-cultural domain, this time concentrating on DVD movies with a sci-fi and fantasy flavor. Again, we found that the reviewers are active within rather narrow bounds. They focus on widely popular titles, overlapping very little with any other possible themes or genres. Only a small part of the reviews by the members of the community are related to something else, e.g., to books by M. Proust and T. Mann.

The third and last example we want to mention is shown in Fig. 8. In analogy with the former examples, the “pop-music” and “pop-movie” communities, we may call this one a “pop-politics” community. The reviewers here concentrate on books discussing the presidency of G. Bush, the evils of liberal ideology, as compared with neo-conservatism, and so on. The core of the community is kept together by the books of Ann Coulter, who is known as a militant anti-liberal writer. Majority of the books in this group is targeted at the widest public, as is the music by The Beatles and movies of the “X-Files” type. Their themes are not esoteric, these products are not aiming at specialized audiences; yet, the zeal of the reviewers makes a “cult” of them. Again, this community is narrowly defined by the interest in these popular issues and not much else. In the “cloud” of other items reviewed by the members of this community we find some other books by Ann Coulter, accompanied by books such as (the titles are self-explaining, we believe) *Worse Than Watergate: The Secret Presidency of George W. Bush*; *Blinded by the Right: The Conscience of an Ex-Conservative*; *A Matter of Character : Inside the White House of George W. Bush*; *The Family : The Real Story of the Bush Dynasty*; *Chain of Command : The Road from 9/11 to Abu Ghraib*, and similar. Out of the six reviewers, only Donnie Brasco shows some additional field of interest, having written about various pop-music CD’s as well.

Let us sum up these observations (supported also by analyses of other small communities we were able to find in the sample). Our expectations that strongly localized eigenvectors would reveal some specific small communities was fulfilled in the sense that we have indeed found groups of zealots, concentrated on a relatively narrow segment of commodities sold on Amazon.com. Individual interests of these reviewers only scarcely reach beyond the theme common to the community.

On the other hand, however, it would be misleading to imagine these people as eccentrics focused on highly specialized, marginal or even extreme cultural artifacts. The subjects of their reviews are quite ordinary, clearly part of the cultural mainstream. And, by their tastes, the reviewers themselves seem belonging to wide audiences, often focused on classics or well-established pop-cultural products. In other words, anomalous tiny fragments of this huge network, characteristic by various authors repeatedly writing reviews on the same items, refer typically not to

some marginal cultural forms with specialized contents, but rather to widely shared cultural tastes and mainstream enthusiasts.

A more detailed analysis of these findings is beyond the scope of this methodological paper and its analytical illustration. Very probably, several possible explanations could turn valid in parallel, including the nature of the Amazon.com portal (primarily designed for general audiences and as wide consumer population as possible), possibly higher probability that reviews on widely favored artifacts get “localized” etc. What should perhaps be stressed here, however, is the peculiar character of the communities or network segments under discussion. It is clear that the tiny network fragments counting 5 or 10 reviewers and dozens of reviews cannot represent “big” consumer populations and “widespread” artifacts in some straightforward way. Rather, they may provide a rather specific (“small-scale”) way of looking at a mass-scale phenomenon. Let us tell something more about this specificity.

We have already noted that the network and its segments we are studying is not a “social network” as traditionally envisaged. The interaction constituting the network is so massively mediated and by-produced (while remaining observable, “real” enough and grounded in intentional social action) that we leave the territory of what is usually counted by social scientists as a “group” or “community”. But even more is at stake in this direction. A closer view of our findings reveals that one cannot unambiguously say whether the “connecting” reviewed products provide interpretive framework for statements about the reviewers, or whether – on the contrary – it is reviewers and their actions that provide clues for interpreting communities of products. In other words, we are unable to determine whether we study groupings of people (connected by products) or of commodities (connected by people). In fact, we should better try to understand both within a single hybrid network, meaningfully connected. While studying phenomena of product reviewing, products and reviewers cannot be separated. The sets of products represented in our figures (Fig. 6 7 8) do not simply make sense (and do not hold together) without the reviews written about them by the represented reviewers. Indeed, the products grouped by, e.g., purchases carried out by Amazon.com users would look differently. On the other hand, the groups of reviewers would not make sense without the particular reviewed products (their amount and nature). Thus, we believe the segments identified in our example can directly represent neither populations of consumers nor entire sections in the Amazon.com commodities catalog. Rather, they represent, in a complex way and as if under a specific lens, a phenomenon of on-line user reviewing, better understanding of which may contribute to our knowledge of contemporary popular culture and technologically mediated economic processes.

6. Conclusions

Thanks to numerous sociological efforts in the field of social network analysis as well as the work on networks done in other scientific disciplines such as theoretical

physics, various mathematical tools have been developed. They aim either at determining large-scale structures in complex networks or at identification of smaller network segments such as cliques or acquaintances. In this work, we introduced a new mathematical procedure relatively close to the latter type of task. We believe the method is well suitable for finding the most relevant small segments of complex networks, when “relevance” cannot or need not be equaled to some absolute level of mutual connectivity between the nodes. We argue that this is often the case, because important social forces or processes are often related to highly mediated and heterogeneous groupings, typically constituted as by-products of various, differently oriented actions, and where people characteristically and usually do not intentionally address each other and even do not know each other (here, we could speak of “ultra-weak” ties). The proposed method based on well-localized eigenvectors is well capable to find these small communities with anomalously high density of mutual links and therefore reveal a kind of semantic information hidden in the network, otherwise often neglected. As such, our method may be a good starting point for more fine-grained further analysis of given phenomena.

As an empirical example, we have chosen the data available from the Amazon.com on-line shopping portal. We studied the network constituted by users writing reviews of the same products offered for purchase on the website during the summer 2005, when the data were gathered. Reviewers become connected if they have written a review on an identical item. When such connections locally proliferate we get a grouping of relevance.

These groupings are not directly related to the top-lists of popularity, but reveal the most focused points in the network. They are constituted by socially rather distant ties, i.e., by a kind of ultra-weak ties, namely highly mediated links by-produced during processes primarily aimed at something else than addressing each other to establish acquaintance or become closer.

The first important result of our analysis is the power-law tail in the density of eigenvalues. This feature is partially, but not entirely, due to the power-law degree distribution. Comparing the spectrum arising from the network with the spectrum of a random network with the same degree sequence, we find a power-law tail in both cases, but the exponent is significantly smaller in the original network. Generally, such a tail implies that the density of cycles beyond certain length diverges when the size of the network tends to infinity. The difference in the exponent means that some shorter cycles keep finite density in the randomized network, while in the original one they are much more abundant. This means that the Amazon network contains much more compact groupings than what would be expected knowing only its degree sequence.

To see at least some of these small groupings, we looked at well-localized eigenvectors. These localized states represent small communities or network segments and bear semantic information hidden in the network. We call them “hot spots”, as they represent local structures which differ from the surrounding background. We were able to explicitly find some of these communities and attribute meaning to

them. The three of them briefly discussed in this paper can be labeled as pop-music, pop-movie, and pop-politics communities. The reviewers of these communities are very strongly focused on one narrow segment. This segment itself belongs usually to mass or popular culture, so it cannot be considered as marginal or esoteric. It is the enthusiasm of the reviewers which singles the segment out of the sea of millions products traded on Amazon.com.

Our analysis shows that only about half per cent of the reviewers belong to these network segments in the small sample of 10^4 reviewers. However, we expect that this fraction would rapidly grow if more reviewers are included from the top of the Amazon list downwards. If carefully treated and interpreted the identified network segments may be useful for enhancing our knowledge of mass or popular culture and complex economic processes related to E-consumerism. Especially, it would be interesting to make systematic classification of the small communities.

Besides these specific findings we would like to highlight another, more general feature. When analyzing the chosen example, it turned out that conventional talking about “networks of reviewers” might be sociologically misleading. Our groupings, in fact, were constituted not only by people writing reviews on the same products, but also (simultaneously) by products reviewed by the same reviewers. That is why we decided to switch to a more appropriate term “networks of reviewing”. This term indicates the hybrid nature of networks we have been dealing with and it allows better talking about processes of online economy rather than on bare structures composed of its human agents. In this respect, our approach is well compatible with the currently increasing emphasis on heterogeneity as an essential quality of collectivities studied by social scientists [60].

The method can be applied in a straightforward way to any kind of network, wherever the data can be collected easily. However, technical limitations of the method may arise in networks larger than several tens of thousands of vertices, due to computer memory limitations. As shown also by the example of Amazon.com, on-line networks are often larger than that. Then, we must decide which subset of the whole network can be considered representative. In our case we chose the subset of the most productive reviewers, but other networks might require other criteria.

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